# 7E Stationary Waves in an Air Column

## Investigation of the harmonics of a vibrating column of air of variable length

### APPARATUS REQUIRED

- resonance tube assembly on a floor stand (see Fig. 7.13 for two versions)
- metre rule
- set of tuning forks (256–512 Hz frequency)
- striking pad or cork bung for striking tuning forks
- beaker of water for topping up resonance tube
- 0–50 °C thermometer

### PRINCIPLES INVOLVED

**a** A tuning fork held above the open end of a tube, as in Fig. 7.12, will result in a longitudinal sound wave travelling down the tube. This wave will be reflected at the closed end of the tube. The superposition of these two waves will establish a stationary wave in the tube. Resonance will only occur for those waves whose wavelengths $\lambda$ are correctly matched to the length of the air column. Then the reflected rarefaction at the open end enters the column at the same instant as the rarefaction from the tuning fork to give constructive superposition. Such waves will give a sound wave of constant intensity.

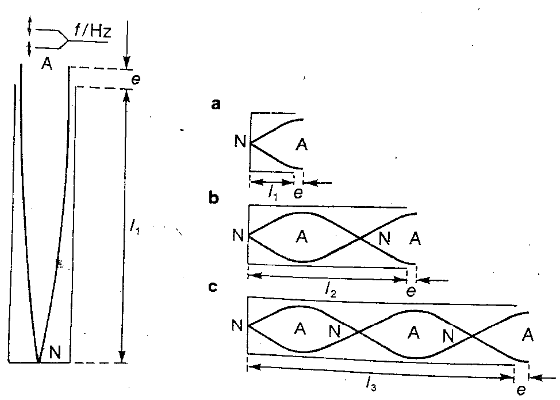


**Fig. 7.12** Displacement curves for an air column subject to forced vibrations: **a** first, **b** second and **c** third resonances.

**b** The boundary conditions at the open end of the tube must correspond to a pressure node and hence a displacement antinode. The boundary conditions at the closed end of the tube correspond to a displacement node and hence a pressure antinode.

**c** A small length of air outside the tube is also set in vibration. The effective length of the vibrating air column exceeds the length of the tube by $e$, the end correction. $e$ is approximately equal to 0.6 times the tube radius.

**d** For the first resonance, Fig. 7.12a, the length of the air column $(l_1 + e) = \lambda/4$. Thus if the speed of the sound wave in the column is $c$:

$$c = 4f(l_1 + e) \qquad [7.5]$$

for a tuning fork of frequency $f$.

Equation [7.5] can be used to investigate the relationship between $f$ and the first resonance length $l_1$, and to determine a value for the speed of sound in damp air.

**e** Further resonances can be obtained for vibrating air columns of length $\frac{3}{4}\lambda$ (Fig. 7.12b) and $\frac{5}{4}\lambda$ (Fig. 7.12c). Thus for the first three resonances:

$$\left.\begin{aligned} l_1 + e &= \tfrac{1}{4}\lambda \\ l_2 + e &= \tfrac{3}{4}\lambda \\ l_3 + e &= \tfrac{5}{4}\lambda \end{aligned}\right\} \qquad [7.6]$$

Subtraction of any of these two equations can be used to find a value for $\lambda$, eliminating the end correction $e$. A further value for $c$ can then be calculated.

### INVESTIGATION OF THE RELATIONSHIP BETWEEN $f$ AND THE LENGTH $l_1$

**f** Set the apparatus up as shown in Fig. 7.13, ensuring that the tube is as near vertical as possible. The procedure given for finding the resonances applies to the apparatus with the movable tube. The procedure for the other version is similar but greater care has to be taken with adjusting the length of the air column.

**g** For the highest frequency tuning fork ($f$ = 512 Hz) calculate an approximate value for $l_1$ using $c$ = 300 m s$^{-1}$. Set $l_1$ to about 3 cm shorter than this length.

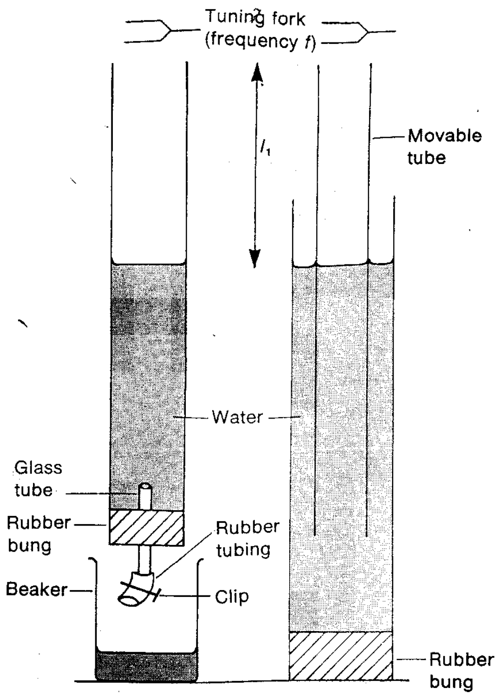


**Fig. 7.13** Measurement of the resonances of a column of air. Two versions of the apparatus are shown.

**h** Carefully adjust the height of the movable tube so as to obtain a maximum in the sound intensity when the 512 Hz tuning fork is struck on its pad and held near to the open end of the tube. Do not strike the fork too hard otherwise it will not vibrate at a single frequency. Measure the length of the air column $l_1$ with a metre rule. Also estimate the uncertainty in $l_1$ by finding the range over which $l_1$ can be varied without significantly altering the sound level.

**i** Tabulate $f$ and $l_1$ (preferably in metres). Leave a further column for data analysis.

**j** Repeat the measurements for a range of frequencies down to 256 Hz. Plot a graph of $l_1$ against $f$ as the experiment proceeds. This will help to identify any doubtful experimental points.

**k** From equation [7.5] choose an appropriate graph to plot so as to obtain a straight line relationship. From this graph determine a value of $c$ from the slope and a value of $e$ from an appropriate intercept. (Hint: If you plot $l_1$ as one of your variables you will need to leave room for a small negative intercept on the $l_1$ axis.)

**l** Correct your value for $c$ to the value of $c$ at 0 °C assuming that $c$ is proportional to the square root of the temperature of the air in kelvin. ($T(\text{K}) = \theta(°\text{C}) + 273$).

Comment on whether the value of $c$ you determined is likely to be lower or higher than the value for dry air.

## INVESTIGATION OF THE FIRST THREE RESONANCES OF AN AIR COLUMN

**m** Because of the limit on the maximum length of the air column (about 1 m) and the decreasing intensity of the resonances, it is only possible to find $l_1, l_2$ and $l_3$ for the highest frequency tuning fork ($f$ = 512 Hz).

**n** Gradually increase the length of the air column and determine the positions of the first three resonances.

**o** Using equation [7.6] determine three values for $\lambda$ by using the equations in pairs. Calculate the average value for $\lambda$ together with an estimate in its uncertainty from your range of $\lambda$ values.

**p** Calculate a value for $c$ from $c = f\lambda$ and correct this value of $c$ to the value for 0 °C as in (l).

## NOTES

**q** Your account should include a detailed account of the measurement of the first resonance for a range of frequencies, and the analysis leading to the value for $c$. Compare your value of $c$ with the accepted value for $c$ in dry air at 0 °C.

**r** Comment on whether the value of $c$ determined graphically in (k) is likely to be more reliable than the value determined in (p).

**s** The principal application of resonance in closed columns of air is in tuning wind instruments to produce notes of a particular frequency.

**t** In principle this experimental arrangement could be used to determine the speed of sound waves in a gas, particularly a gas that is denser than air.