c (*Group discussion*)

(i) How realistic is the above scenario?

(ii) How could it be simply adjusted to cater for multiple pets?

8 Sixty per cent of individuals of one species of native tree are known to have white flowers, the rest pink. The trees are further classified as having single flowers or double flowers. A survey taken of a forest of these trees revealed that 25% of the white-flowering trees and 40% of the pink-flowering trees had single flowers.

Determine the probability that one of these trees, selected at random, will be

a white double-flowered **b** single-flowered

c pink single-flowered **d** white or double-flowered.

9 Of 1000 consumers trialling a particular brand of prewash spray, 157 have reported a discolouration of cotton items.

a The manufacturing company has decided to ask three members of its staff to trial the spray. Determine the probability that

(i) all three will find that cotton items become discoloured after they use the spray

(ii) none will find that cotton items become discoloured after they use the spray.

b Would you use this spray? Justify.

c A company has asked a further

(i) 7 (ii) 13 (iii) 30

staff members to trial the spray. Discuss how these numbers of trials would affect your working of this question.

5.6 Binomial expansions

When we multiply two numbers together, e.g. 21×43, we multiply 1 by 3, 2 by 3, 1 by 4, 2 by 4 and tidy up.

$$\begin{array}{r} 21 \\ \times\ 43 \\ \hline 63 \\ 84 \\ \hline 903 \end{array}$$

In algebra, we multiply the same way, e.g. for $(x + y)(x + y)$, we multiply

y by y $[= y^2]$ y by x $[= xy]$

x by y $[= xy]$ x by x $[= x^2]$

and tidy up:

$$y^2 + xy + xy + x^2$$
$$= y^2 + 2xy + x^2$$

or

$(x + y)(x + y)$ (Arrows show the multiplications.)

$= x^2 + xy + xy + y^2$

$= x^2 + 2xy + y^2$

	x	$+ y$
	x	$+ y$
	xy	y^2
x^2	yx	
$x^2 + 2xy + y^2$		

Just as we can't multiply three **numbers** together in one step, multiplying in algebra seems difficult when there are more than two brackets. However, it is in fact quite simple!

Consider Pascal's triangle. (This was first used by the French mathematician Blaise Pascal in the seventeenth century.)

Each row starts and finishes with 1.

Each other number is determined by adding the two above it.

```
                1
              1   1
            1   2   1
          1   3   3   1
        1   4   6   4   1
      1   5  10  10   5   1
```

Pascal's triangle

Consider $(x + y)^4$. To multiply this out (or expand it), we follow these steps.

Step 1

Write down the powers of x in descending order. Start at 4 (since the overall power is 4) and leave gaps.

$x^4 \quad x^3 \quad x^2 \quad x^1 \quad x^0$

Step 2

Beside these, write down the powers of y in ascending order. Start at 0.

$x^4y^0 \quad x^3y^1 \quad x^2y^2 \quad x^1y^3 \quad x^0y^4$

Step 3

Select the appropriate row from Pascal's triangle (the overall power is 4, therefore the appropriate row has 4 as the second number).

$1x^4y^0 \quad 4x^3y^1 \quad 6x^2y^2 \quad 4x^1y^3 \quad 1x^0y^4$

Step 4

Omit the ones, the 0 powers, and add + signs, to get the final answer.

$x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

Exercise 5.6

1 Expand the following.

a $(x + y)^5$ **b** $(x + y)^3$ **c** $(a + b)^5$ **d** $(a + b)^2$

e $(p + q)^6$ **f** $(p + q)^5$ **g** $(m + n)^4$ **h** $(m + n)^7$

i $(s + t)^9$ **j** $(s + t)^{10}$

2 **a** Write down the first five lines of Pascal's triangle.

b Using your calculator, work out the following.

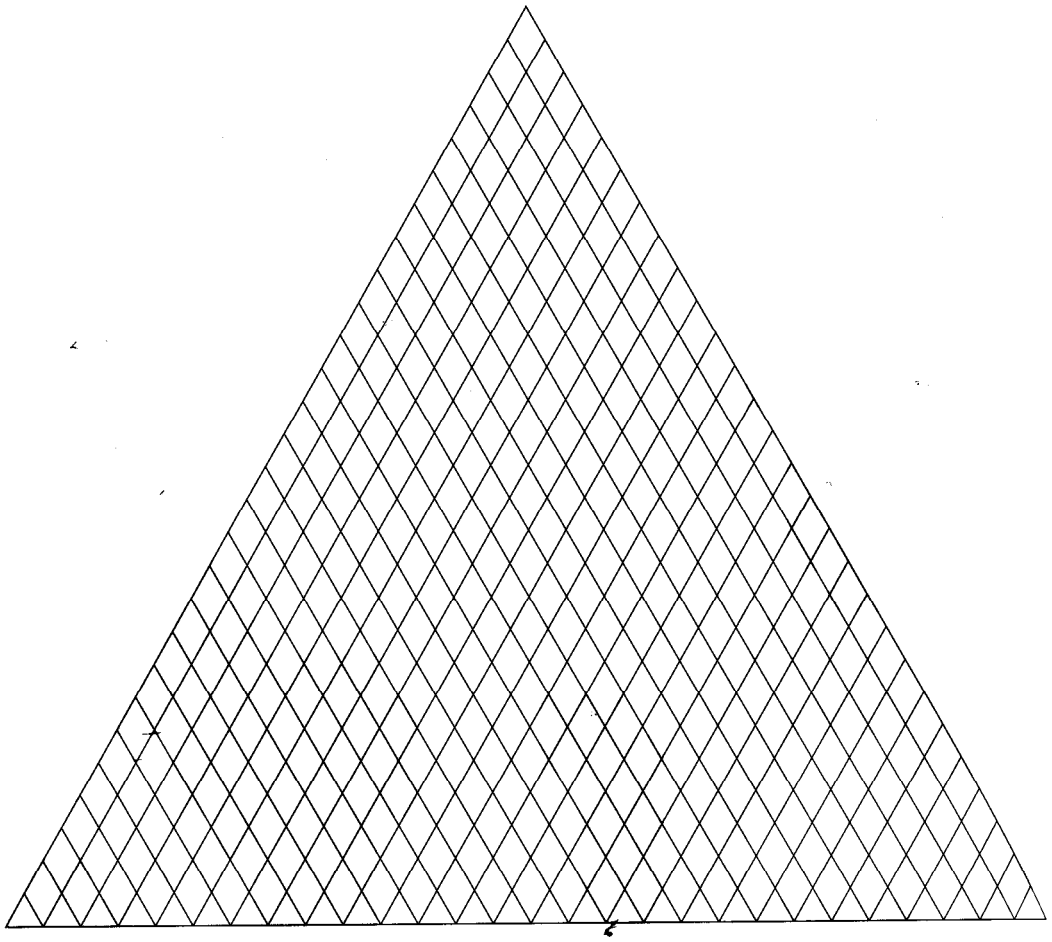
(i) 11^0 (ii) 11^1 (iii) 11^2

(iv) 11^3 (v) 11^4

c Compare your answers with Pascal's triangle.

d Sum the numbers in each line of Pascal's triangle. What do you notice?

3 (*Group work*) Copy the diagram below of a triangle divided into diamonds.



a Fill in the diamonds with Pascal's triangle.

b Each member of the group then chooses a different number, e.g. 2, 3, 4, 5 etc. Colour in each segment containing a multiple of the number you have chosen.

c Compare patterns.

4 Use the table of binomial coefficients (see Appendix, page 524) to repeat question **1**.

5.7 Binomial Theorem for probability

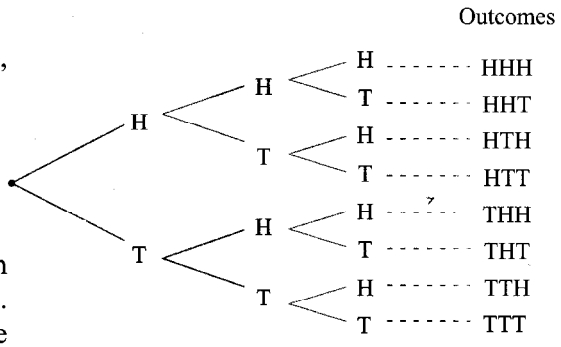
Consider a coin tossed three times.

If we write HHH as H^3 and HHT as H^2T etc., we have the outcomes

H^3, H^2T, H^2T, HT^2, H^2T, HT^2, HT^2, T^3.

i.e. $H^3 + 3H^2T + 3HT^2 + T^3$

i.e. the binomial expansion of $(H + T)^3$.



We could use either the binomial expansion or the tree diagram for this type of question. *But*, if we tossed the coin 20 times, the tree diagram would get too messy! (We saw this in question **11** of exercise 5.4.)

Under certain circumstances, we can use a binomial expansion, also called the Binomial Theorem, to determine the probability of an outcome.

Conditions for using the Binomial Theorem

1. The same experiment must be **repeated** a number of times (under the same conditions), e.g. toss a coin six times.
2. There can only be **two outcomes** in each trial (we call them success or failure), e.g. seeds grow or die.
3. Each trial must be **independent**, i.e. no trial must depend on the previous trial.

Using the Binomial Theorem for probability

1. Select the two outcomes. Choose one as success (p) and the other as failure (q).
2. Assign a probability to p and q, on **one** trial.
3. Choose the power to equal the number of trials.
4. Expand binomially.
5. Evaluate the term required.

Worked example

A producer of flower bulbs guarantees that 85% of gladioli bulbs sold will germinate (grow). A gardener plants ten of the bulbs. What is the probability that

a none will grow

b six will grow

c at least nine will grow?

Check

1 The same event is repeated with each bulb.

2 There are only two outcomes: each bulb will either grow or die.

3 The growth of each bulb does not depend on the other bulbs.

Therefore we can use the Binomial Theorem.

Setup

Step 1

success = germinate = p

failure = no grow = q

Step 2 (for one bulb)

$p = 0.85$
$q = 0.15$ (The probabilities must add up to 1.)

Step 3 (choose the power)

10 bulbs are planted, therefore power is 10

Step 4 (expand binomially)

$$(p + q)^{10} = p^{10} + 10p^9q + 45p^8q^2 + 120p^7q^3 + 210p^6q^4 + 252p^5q^5 + 210p^4q^6 + 120p^3q^7 + 45p^3q^8 + 10pq^9 + q^{10}$$

Calculate

Step 5 (evaluate the term required)

a $P(0 \text{ successes}) = q^{10}$
$= (0.15)^{10}$
$= 0.000\,000\,005\,8$

Thinking:
0 successes, 10 failures
p is success, q is failure
$\therefore$ we want p^0 or q^{10}

b $P(6 \text{ successes}) = 210p^6q^4$
$= 210 \times (0.85)^6 \times (0.15)^4$
$= 0.0401$

Thinking:
6 successes
p is success
$\therefore$ we want p^6
whole term is $210p^6q^4$

c $P(9 \text{ or more successes}) = p^{10} + 10p^9q$
$= (0.85)^{10} + 10 \times (0.85)^9 \times (0.15)$
$= 0.5443$

Thinking:
9 or more means 9 or 10
we want p^9 or p^{10}

Exercise 5.7

1 Statistics show that more male babies are born than female. Assume that the probability of a male child being born is $\frac{3}{5}$. Consider a family of 6 children.

a Check

(i) Are there only two outcomes?

(ii) Are successive births independent?

(iii) Can we use the binomial theorem?

b Setup

(i) Select the two outcomes, and label as p and q (you may use s and f if you prefer).

(ii) Assign a probability to each.

(iii) Determine which power to use.

(iv) Expand binomially.

Calculate

c Calculate the probability of a couple having a family of

(i) three boys and three girls

(ii) four girls

(iii) at least one boy.

d Research families in your school with six children to determine if this binomial distribution is a suitable model for real life. If your sample size is too small, choose a different-sized family, e.g. three children or four children.

2 Assume that wattle trees of a particular species have a 30% chance of survival when transplanted. Ten wattle trees are transplanted to a school forestry plot.

a Check whether the conditions are suitable for using the Binomial Theorem.

b Set up to use the Binomial Theorem.

c Determine the probability of the following results.

(i) No trees will survive.
(ii) Six trees will survive.
(iii) At least eight trees will survive.
(iv) Less than three trees will survive.
(v) Five or six trees will grow.

3 The school netball team has a history of winning three out of every four games they play. In a round robin competition, they are required to play five games. Calculate the probability of the following results.

a The team wins all its games in the competition.

b The team wins only one game.

c The team wins more than three games.

d The team loses two games.

e The team loses only one game, and wins four, thus winning the competition.

4 Quality assurance controllers at a clothing factory have found that 10% of the tracksuits manufactured have faulty seams. A school has purchased seven of these tracksuits for their cheer squad.

What is the probability of the outcomes in **(a)**, **(b)** and **(c)**?

a At least half the cheer squad have faulty seams in their tracksuits.

b All the tracksuits purchased have to be returned.

c None of the tracksuits have faulty seams.

d Comment on whether this level of production of faulty garments is acceptable. Should the school change its supplier?

5 An analysis of medical data has shown that approximately 5% of the male population is colour blind. In a group of six boys, calculate the probability of the following.

a All are colour blind.
b Three are colour blind.
c At least four are colour blind
d None are colour blind.

6 A club sells Lucky Envelopes to raise money, and guarantees that one in every ten wins a prize. The tickets are sold in random order. A patron buys six tickets. Find the probability that he wins:

a no prizes **b** at least three prizes **c** five prizes.

7 A science exam has a multiple choice section of ten questions requiring a true or false answer. Ben has not studied, and guesses the answers. What is the probability that he gets:

a all questions correct

b at least eight out of ten correct

c less than three correct

d a 'pass', i.e. five or more correct

e a 'high' on this paper, i.e. at least six but fewer than eight correct?

8 Another science exam paper has ten multiple choice questions with four possible answers for each question, only one of which is correct. If a student guesses each answer, what is the probability she will get

a all questions correct

b at least seven out of ten correct

c less than three correct

d a 'pass', i.e. five or more correct

e a 'limited' on this paper, i.e. at least three but less than five correct?

9 A certain group of pilots are known to be on target (i.e. within a 50 m radius of a drop zone) 75% of the time. One of their planes sets out to drop six food bundles to people stranded by floodwaters. (Each bundle contains ten bags of food.) Calculate the probability of the following outcomes.

a All bundles miss the target zone completely.

b At least 40 bags of food reach the stranded people.

c Only two food bundles arrive on target.

d The stranded people have enough food to last five days (they need at least eight bags of food per day).

10 Statistics collected in a survey reveal that only eight out of ten houses in a particular suburb were connected to pay TV. In a street of eight houses in this suburb (all of which were included in the survey) calculate the probability that

a none of these eight houses selected for the survey had pay TV

b at least half had pay TV

c less than three houses were connected to pay TV

d all of the houses were connected to pay TV.

11 Before new flavours of potato chips are released onto the market, trials must show that the flavour has an 80% approval rate. A new flavour has just been released onto the market and a teacher decides to trial the chips with ten of her students as part of a market research project. Calculate the probability that

a all ten students will like the new flavour

b at least eight of the students will like the flavour

c none of the students will like the flavour

d at least three of the students will like the flavour.

12 A seed producer guarantees that, if a particular kind of seed is planted under the correct conditions, 90% of the seeds will germinate. If 12 seeds are planted according to the directions, calculate the probability of the following outcomes.

a At least nine seeds germinate.

b Less than three seeds germinate.

c At least 50% of the seeds germinate.

Exercise 5.3

1 **a** $\frac{1}{5}$ **b** $\frac{1}{100}$ **c** $\frac{1}{500}$ **d** $\frac{2}{5}$ **2** **a** $\frac{11}{850}$ **b** $\frac{2}{17}$ **c** $\frac{1}{5525}$ **d** $\frac{11}{41\,650}$
3 (With replacement)

a $\frac{1}{27}$ **b** $\frac{8}{243}$ **c** $\frac{32}{243}$ **d** $\frac{16}{81}$

(Without replacement)

a $\frac{1}{84}$ **b** $\frac{1}{21}$ **c** $\frac{1}{7}$ **d** $\frac{2}{7}$ **4** **a** $\frac{1}{1\,000\,000}$ **b** $\frac{1}{1\,000\,000}$ **c** $\frac{1}{10}$ **d** $\frac{1}{1\,000\,000}$
e $\frac{1}{100}$ **f** $\frac{1}{50\,000}$ **g** $\frac{1}{50}$ **5** **a** (i) $\frac{1}{12}$ (ii) $\frac{1}{66}$ (iii) $\frac{1}{660}$ (iv) $\frac{1}{11\,880}$ (v) $\frac{1}{22}$
b JHH, HJH **c** 0.076
d No pairs then W, C

WC
– WC
– – WC
– – – WC
– – – – WC
– – – – – WC

e 0.030 78 **6** **a** (i) 0.000 000 245 (ii) 0.2975 (iii) 0.021 419 6 (iv) 0.005 327 98
(v) 0.197 350 6 (vi) 0.2892 **b** (i) 0 (ii) 0.003 007 5 (iii) 0.000 294 9 **c** Cockatoo
d Platypus

Exercise 5.4

1 **b** $\frac{1}{4}$ **c** $\frac{1}{4}$ **d** $\frac{1}{2}$ **e** $\frac{1}{4}$ **f** $\frac{1}{4}$ **g** $\frac{3}{4}$ **2** **b** $\frac{1}{8}$ **c** $\frac{3}{8}$ **d** $\frac{7}{8}$ **e** $\frac{1}{2}$
f $\frac{1}{4}$ **g** $\frac{1}{8}$ **3** **a** $\frac{1}{4}$ **b** $\frac{3}{8}$ **c** $\frac{1}{16}$ **d** $\frac{15}{16}$ **e** $\frac{15}{16}$ **f** $\frac{1}{2}$ **5** **b** $\frac{1}{6}$ **c** $\frac{5}{18}$
d $\frac{5}{18}$ **e** $\frac{11}{36}$ **f** $\frac{1}{12}$ **g** $\frac{5}{12}$ **h** $\frac{1}{2}$ **i** $\frac{1}{18}$ **j** $\frac{1}{4}$ **k** $\frac{1}{36}$ **6** **a** $\frac{1}{5}$ **b** $\frac{4}{25}$
c $\frac{1}{5}$ **d** $\frac{3}{25}$ **e** $\frac{4}{25}$ **f** $\frac{2}{25}$ **7** **a** 5×5 **b** 9 **c** 2–10 **d** 2, 3, 4, 8, 9, 10 **e** 2
f 6 **8** **c** Yes **d** $\frac{2}{5}$ **e** (i) $\frac{1}{10}$ (ii) $\frac{1}{10}$ **f** (i) 0 (ii) $\frac{1}{10}$ (iii) $\frac{2}{5}$ (iv) $\frac{1}{5}$
(v) 0

Exercise 5.5

1 **a** 0.25 **c** (i) 0.375 (ii) 0.0625 (iii) 0.4375 (iv) 0.4375 **e** (i) 0.4219
(ii) 0.75 (iii) 0.015 625 (iv) 0.046 875 **2** **a** (i) 0.077 76 (ii) 0.2304 (iii) 0.0768
(iv) 0.68256 (v) 0.33696 (vi) 0 **3** **a** 0.33642 **b** 0.005252 **c** 0.2352 **d** 0.116029
4 **a** 0.3164 **b** 0.9492 **c** 0.738 28 **d** 0.996 09 **e** 0.3164 **f** 0.738 28 **5** **a** (i) 0.18975
(ii) 0.882 87 (iii) 0.013 36 **6** **a** 0.05 **b** 0.475 **c** 0.31667 **d** 0.15833
e Insufficient information **7** **b** (i) 0.3333 (ii) 0.3333 (iii) 0.8
(iv) Insufficient information (v) 0.5333 **8** **a** 0.45 **b** 0.31 **c** 0.16 **d** 0.84
9 **a** (i) 0.00387 (ii) 0.5991

Exercise 5.6

1 **a** $x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$ **b** $x^3 + 3x^2y + 3xy^2 + y^3$
c $a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$ **d** $a^2 + 2ab + b^2$
e $p^6 + 6p^5q + 15p^4q^2 + 20p^3q^3 + 15p^2q^4 + 6pq^5 + q^6$ **f** $p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4 + q^5$
g $m^4 + 4m^3n + 6m^2n^2 + 4mn^3 + n^4$ **h** $m^7 + 7m^6n + 21m^5n^2 + 35m^4n^3 + 35m^3n^4 + 21m^2n^5 + 7mn^6 + n^7$
i $s^9 + 9s^8t + 36s^7t^2 + 84s^6t^3 + 126s^5t^4 + 126s^4t^5 + 84s^3t^6 + 36s^2t^7 + 9st^8 + t^9$
j $s^{10} + 10s^9t + 45s^8t^2 + 120s^7t^3 + 210s^6t^4 + 252s^5t^5 + 210s^4t^6 + 120s^3t^7 + 45s^2t^8 + 10st^9 + t^{10}$

Exercise 5.7

1 **a** (i) Yes (ii) Yes (iii) Yes **b** (i) $p = \frac{3}{5}, q = \frac{2}{5}$ (iii) 6
(iv) $p^6 + 6p^5q + 15p^4q^2 + 20p^3q^3 + 15p^2q^4 + 6pq^5 + q^6$ **c** (i) 0.276 48 (ii) 0.138 24 (iii) 0.9959
2 **c** (i) 0.028 25 (ii) 0.036 76 (iii) 0.001 59 (iv) 0.3828 (v) 0.1397
3 **a** 0.2373 **b** 0.014 65 **c** 0.6328 **d** 0.2637 **e** 0.3955 **4** **a** 0.002 728
b 0.000 000 1 **c** 0.4783 **5** **a** 0.000 000 015 625 or 1.5625×10^{-8} **b** 0.002 143 4
c 0.000 086 4 **d** 0.735 09 **6** **a** 0.531 44 **b** 0.015 85 **c** 0.000 054 **7** **a** 0.000 976 6
b 0.054 69 **c** 0.054 69 **d** 0.623 05 **e** 0.322 27

Note that answers are given in two forms for questions **8–19** (see the repeat set of answers below.)

8 **a** 0.000 000 953 7 **b** 0.003 506 **c** 0.525 59 **d** 0.078 127 **e** 0.396 28
9 **a** 0.000 244 14 **b** 0.830 57 **c** 0.032 96 **d** 0.830 57 **10** **a** 0.000 002 56 **b** 0.989 59
c 0.001 231 4 **d** 0.167 77 **11** **a** 0.107 37 **b** 0.677 799 5 **c** 0.000 000 102 4
d 0.999 922 **12** **a** 0.974 36 **b** 5.455×10^{9} **c** 0.999 95 **d** 0.000 0467 67 **e** 0.110 87
13 **a** 0.275 897 **b** 0.179 804 **c** 0.196 87 **d** 5.767×10^{-9} **e** 0.998 617 **14** **a** 0.042 47
b 0.167 29 **c** 0.000 104 9 **d** 0.953 64 **e** 0 **15** **a** 0.137 09 **b** 0.108 643
c 0.299 85 **d** 0.006 225 97 **e** Not enough information **f** 0.569 37 **16** **a** 0.107 37
b 0.005 505 **c** 0.879 13 **d** 0.879 13 **e** 0.026 42 **17** **a** 0.272 49 **b** 0.018 499 **c** 0.894 79
d 0.105 213 **18** **a** 0.082 305 **b** 0.329 218 **c** 0.680 38 **d** 0.087 79 **19** **a** 0.000 031 09
b 1.134×10^{-8}

Answers for **8–19** could be expressed to four decimal places, as follows.
8 **a** 9.537×10^{-7} **b** 0.0035 **c** 0.5256 **d** 0.0781 **e** 0.3963 **9** **a** 0.0002
b 0.8306 **c** 0.0330 **d** 0.8306 **10** **a** 2.56×10^{-6} **b** 0.9896 **c** 0.0012 **d** 0.1678
11 **a** 0.1074 **b** 0.6778 **c** 1.024×10^{-7} **d** 0.9999 **12** **a** 0.9744 **b** 5.455×10^{-9}
c 0.99995 **d** 4.677×10^{-5} **e** 0.1109 **13** **a** 0.2759 **b** 0.1798 **c** 0.1969
d 5.767×10^{-9} **e** 0.9986 **14** **a** 0.0425 **b** 0.1673 **c** 0.0001 **d** 9.5364×10^{-1} **e** 0
15 **a** 0.1371 **b** 0.1086 **c** 0.2999 **d** 0.0062 **e** Not enough information **f** 0.5694
16 **a** 0.1074 **b** 0.0055 **c** 0.8791 **d** 0.8791 **e** 0.0264 **17** **a** 0.2725
b 0.0185 **c** 0.8948 **d** 0.1052 **18** **a** 0.0823 **b** 0.3292 **c** 0.6804 **d** 0.0878
19 **a** 3.109×10^{-5} **b** 1.134×10^{-8}

Exercise 5.8

1 **a** 0.0002 **b** 0.1643 **c** 0.0277 **d** 0.2344 **e** 0.0011 **f** 0.0002 **g** 0.0002
h 0.2503 **i** 0.0661 **j** 0.0000 **k** 0.0887 **l** 0.0874 **2** **a** 0.0131 **b** 0.2252
c 0.0393 **d** 0.0007 **e** 0.0008 **3** **a** 0.0002 **b** 0.1028 **c** 0.0011 **d** 0.4049
e 0.9999 **4** **a** 0.0222 **b** 0.0020 **c** 0.0000 **d** 0.794 **e** 0.0322 **5** **a** 0.0917
b 0.0393 **c** 0.3134 **d** 0.0008 **e** 0.0008 **6** **a** 0.0573 **b** 0.9804 **c** 0.9972
d 0.8418 **7** **a** 0.0515 **b** 0.0425 **c** 0.0011 **8** **a** 0.1298 **b** 0.3771 **c** 0.0115
d 0.0001 **e** 0.8298

Exercise 5.9

1 **a** 1258 **b** 8, 19, 23 **c** 39, 6, 27

d $P(8) = \frac{134}{1258}$

$P(19) = \frac{126}{1258}$

$P(23) = \frac{125}{1258}$

$P(39) = \frac{92}{1258}$

$P(6) = \frac{96}{1258}$

$P(27) = \frac{96}{1258}$