Using Tree Rewiring to Study “Edge” Organisations for C2

Dr. Anthony H. Dekker
Defence Science and Technology Organisation, Canberra, Australia
dekker@ACM.org

Abstract. In recent times, there has been considerable interest in the potential advantages of decentralised “Edge” organisations. In order to investigate the potential costs and benefits of such organisational structures for Command and Control (C2), we have combined organisational simulation with a novel tree rewiring process that creates a spectrum of organisational structures ranging from a pure hierarchy to a pure “Edge” organisation. This paper discusses our simulation framework, presents experimental results, and discusses possible implications for C2 in military forces. The tree rewiring process allows us to analyse organisational structures which are intermediate between a pure hierarchy and a pure “Edge” organisation, and is controlled by a rewiring parameter between 0 and 1. For small values of the parameter, the rewiring process adds informal cross-links to the hierarchy, while for large values of the parameter, the original hierarchical structure is also broken down. The experimental results show that performance is best for completely random “Edge” networks, but that when we take into account link costs and differences in perceived information usefulness for different parts of the organisation, the optimal structures are intermediate between a hierarchy and a pure “Edge” organisation. However, social and technical mechanisms to increase the flow of information will improve this relatively poor optimum, and move it in the direction of greater “Edge” orientation.

1. INTRODUCTION

Following the publication of the influential book by David Alberts and Richard Hayes [1], there has been substantial interest in “Edge” organisations [2],[3]. Such organisations seek to empower individuals and encourage emergent leadership at their periphery.

As part of such a strategy of empowerment, Alberts and Hayes introduce the term “edge,” with its connotations of “cutting edge” or “sharp edge.” They do not use the more traditional (hierarchically-oriented) word “bottom,” which disempowers individuals at the “edge” of an organisation, through its implications of low status and lack of decision-making ability. Similarly, the use of the word “centre” for senior staff, in preference to the hierarchically-oriented word “top,” helps to empower individuals at the “edge.”

An “Edge” organisation often needs to retain a hierarchy in order to transmit the initial command intent from the “centre.” However, empowering the “edge” of an organisation supports self-synchronisation in the execution of that intent. Such self-synchronised execution is further supported by an organisational structure which is less hierarchical and better connected. The extreme example of such a non-hierarchical organisation would be one where individuals were connected at random without regard to rank or status.

Influenced by the NATO SAS-050 working group, and the C2 Conceptual Reference Model which it produced, Alberts and Hayes have produced a more recent book on Command and Control [2], which suggests that there are three key factors which define the essence of C2:

- the allocation of decision rights;
- patterns of interaction among the actors; and
- the distribution of information.

Classic hierarchical organisations have:

- unitary decision rights;
- tightly constrained patterns of interaction; and
- tight control of information distribution.

In contrast, “Edge” organisations have:

- decentralised decision-making;
- unconstrained patterns of interaction; and
- broad dissemination of information.

In this paper, we introduce a tree rewiring process which interpolates between these two extremes, and we test the performance of the resulting spectrum of organisations using a simple simulation model. The tree rewiring process addresses the last pair of this triplet of C2 characteristics (patterns of interaction and information dissemination), and does so jointly.

In other work [3], we have addressed the issue of decision rights, and the circumstances under which decentralised decision-making is appropriate. In the discussion of the experiments reported here, we also summarise some of those findings.

2. TREE REWIRING

We have developed a tree rewiring process that transitions between hierarchical tree structures (such as that in Figure 1), and random “Edge” organisations. Our process is inspired by the network rewiring process of Duncan Watts [4], which transitions between regular and random networks by randomly rearranging links with a probability p, such that $0 \leq p \leq 1$. The Watts process is, however, not suitable for rewiring trees, since such rewiring would disconnect the tree structure. In addition, “Edge” organisations are intended to be robustly networked, and so the transition from a tree structure to an “Edge” organisation should add additional links as well as rearranging the structure.

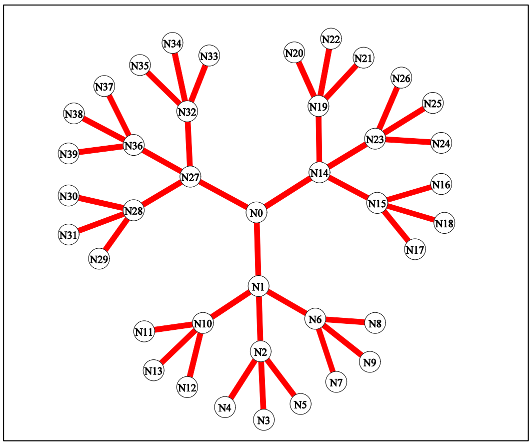

Figure 1: Hierarchical tree structure, with the commanding agent shown in the centre

Our tree rewiring process is controlled by two parameters, a **rewiring parameter** $\boldsymbol{q}$, and a **link density** $\boldsymbol{\delta}$. Each link in the original hierarchy is treated as if it were a “bundle” of δ links. Each of the links in the “bundle” is rewired with probability p, where:

$$p = \sqrt[\delta]{q} \qquad (1)$$

If all the links in the “bundle” are rewired, then the original link vanishes. The probability of this occurring is $p^{\delta} = q$.

The probability of losing a hierarchical link is thus simply the rewiring parameter q, and so, for a tree with n nodes, the expected number of remaining hierarchical links is H, where:

$$H \approx (n-1)(1-q) \qquad (2)$$

For each original hierarchical link, additional links are created, the expected number of which is $p\delta$. Table 1 shows the value of this product for various choices of δ and q. The total number of additional links created is A, where:

$$A \approx (n-1)\,p\delta \qquad (3)$$

The total number of links in the rewired network is therefore given by A+H, where:

$$A + H \approx (n-1)(1 + p\delta - q) \qquad (4)$$

Table 1: Expected number of links created (per original hierarchical link) during tree rewiring

		Rewiring Parameter (*q*)									
		1	**0.9**	**0.75**	**0.5**	**0.2**	**0.1**	**0.05**	**0.02**	**0.01**	**0**
Link Density	**2**	2	1.9	1.7	1.4	0.9	0.6	0.4	0.3	0.2	0
	3	3	2.9	2.7	2.4	1.8	1.4	1.1	0.8	0.6	0
	4	4	3.9	3.7	3.4	2.7	2.2	1.9	1.5	1.3	0
	5	5	4.9	4.7	4.4	3.6	3.2	2.7	2.3	2.0	0

Figure 1 shows the initial hierarchy that we have used for our experiments, which has n = 40 nodes. Figure 2 shows the impact of a link density δ = 5, together with the maximum rewiring parameter q = 1. Here all the original hierarchical links have been lost, but each such link has been replaced by 5 randomly placed other links. The network in Figure 2 represents a highly-connected form of “Edge” organisation.

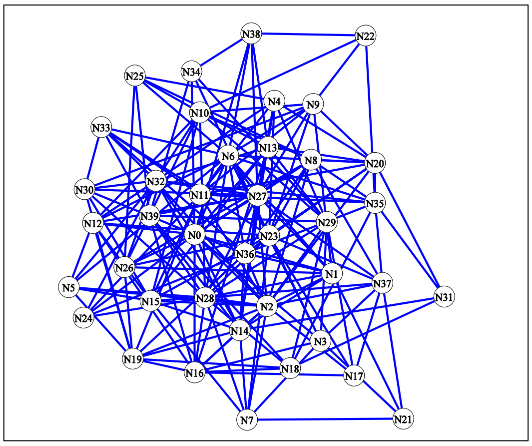

Figure 2: Highly-connected “Edge” network, created with δ = 5 and q = 1

Figure 3 shows a network with link density δ = 4 and rewiring parameter q = 0.1, intermediate between a hierarchy and an “Edge” organisation. Here about 10% of the original hierarchical links have been lost, but many more new links have been added.

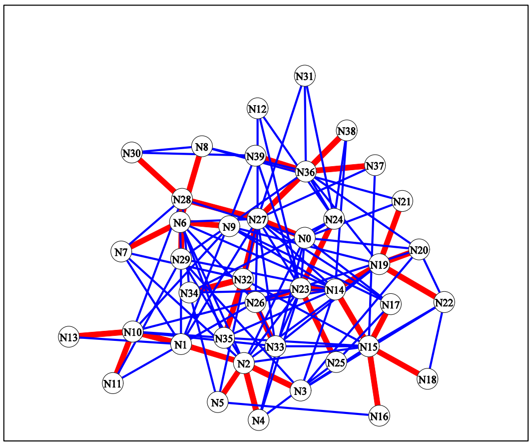

Figure 3: Intermediate network, created with δ = 4 and q = 0.1

In summary, the rewiring parameter q controls how much of the original hierarchical structure is lost, and the link density δ together with q controls how many new links are created.

3. THE LAGRANGE MODEL

For this study, we use a simple model of collaborative problem-solving in organisations (previously introduced in [5]), in which a network of agents each begin with a single piece of data, modelled by a data-number in the range 0…39.

Every agent also has a problem to solve, modelled by a target-number in the range 9…48. The agent solves the problem by finding four data-numbers, all of which are perfect squares, and which add up to the target-number, e.g. 25 + 0 + 4 + 4 = 33. By Lagrange's Theorem [6], such a set of data-numbers can always be found. Agents are not able to compute data-numbers by subtraction: they can only receive data-numbers, add them together, and compare the totals to their target-number. Consequently, agents must collaboratively exchange information with each other over the network in order to solve their individual problems.

There is no distinction between "manager" and "worker" agents: all agents in the network seek to solve their individual problem, and exchange information with other connected agents.

Agents begin by broadcasting their initial data-numbers to their neighbours. At each time-step following, agents broadcast a randomly chosen data-number from a list of those which they have found useful. Usefulness in this case means they have used it to make progress on solving their problem, by creating new partial sums like 25 + 1 = 26. Not being able to subtract, the agents will not know if these data numbers will in fact form part of their final solution.

4. FIRST EXPERIMENT

In our first experiment, we began with a hierarchical network of agents, structured as in Figure 1. Associated with each agent was a data-number in the range 0…39, and a target-number in the range 9…48. The potentially useful data-numbers were therefore 0, 1, 4, 9, 16, 25, and 36.

For each combination of link density δ and rewiring parameter q in Table 1, we performed 100 simulated runs, thus collecting 4000 data points.

In previous experiments [7], the **average distance *D*** between nodes was a useful predictor of performance. We therefore calculated this metric for each of the networks produced. It ranged from 1.8 to 4.4 and was highly correlated with the total number of links $A+H$. The correlation was 0.997 ($R^2 = 99.5\%$), and the line of best fit was:

$$D \approx 1.2 + \frac{124}{A+H} \quad (5)$$

Other network metrics were also highly correlated with the average distance D. For example, the clustering coefficient [4] C was well-predicted (with a correlation of 0.93, $R^2 = 87\%$) by:

$$C \approx \frac{0.95}{D} - 0.23 \quad (6)$$

Agents do not always solve their problems. If they do so, it is always within 1000 time-steps. For each of the 4000 simulation runs, we therefore terminated execution at 1000 time-steps, and recorded an **adjusted completion time *T***, which was:

- the time t, if all agents solved their problems in $t \leq 1000$ time-steps; or
- the product 1000 $(f + 1)$, if at 1000 time-steps, $f \geq 1$ agents had their problem still unsolved.

This adjusted completion time incorporates a penalty for agents failing to solve their problems. The penalty is defined so that successful runs (where all agents solved their problems) always have smaller adjusted times than runs where one or more agents failed to solve their problems. The adjusted completion time therefore measures the time taken for agents to complete their tasks (if they all finish), or (using the penalty) the number of agents which fail to do so. The second factor is in fact dominant in the results.

The adjusted completion time T was best predicted by the average distance D, or rather, by its square, with a correlation of 0.97 ($R^2 = 94\%$). The line of best fit was:

$$T \approx 1690\, D^2 - 5350 \quad (7)$$

Figure 4 shows the scatter-plot associated with this line of best fit.

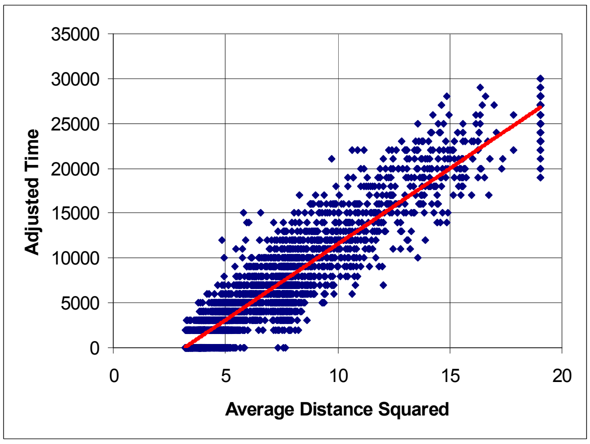

Figure 4: The square of the average distance D between nodes predicts T with a correlation of 0.97

The need for the quadratic in this line of best fit is statistically significant (at the 0.005 level, by analysis of variance), and higher powers or other variables do not improve the prediction. However, given that other variables such as the clustering coefficient are highly correlated with the average distance D, we cannot be certain if the average distance is the only factor controlling performance, or if network connectivity properties [8] have an independent impact.

Figure 5 shows the average values of the adjusted completion time T for each combination of link density δ and rewiring parameter q in Table 1. The best performance was obtained with $\delta = 5$ and $q = 1$, as in the network in Figure 2. The average adjusted completion time T for this case was 246 (with a standard deviation of 651). The high standard deviation for this case reflects a strongly skewed distribution, due to the definition of T. However, overall the values of T are approximately normal (i.e. have moderately low skew and kurtosis), thus allowing standard statistical techniques to be used.

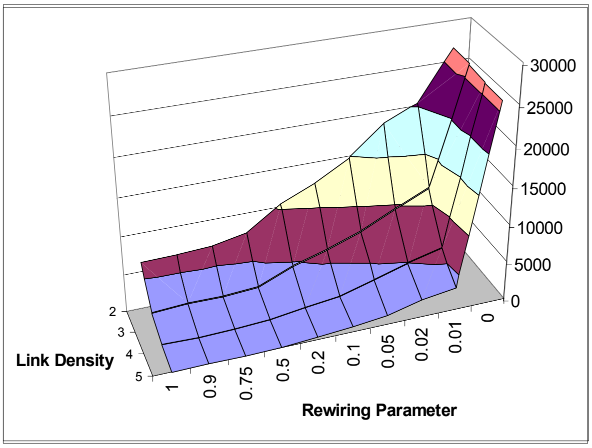

Figure 5: Average adjusted completion times T for rewired networks

5. INCORPORATING LINK COSTS

While unlimited connectivity may be the best in theory, in practice increased networking comes with a cost. Adding new communications infrastructure has an obvious dollar cost, and increased information flows may have organisational costs, such as information overload.

We model this by including link costs. When we add a penalty of 60 time units per link to the adjusted completion time T, we obtain the graph in Figure 6. Here the best performance was obtained either when $\delta = 3$ and $0.5 \le q \le 1$, or when $\delta = 4$ and $q = 0.1$ (the case in Figure 3). Similar results are obtained with different penalty values, but the penalty value of 60 was chosen so that the minima in Figure 6 are just under 10,000 and hence more clearly visible in the graph.

There was no statistically significant difference between these cases. The average value of the cost-adjusted completion time $T + 60(A + H)$ for these cases was 9580 (with a standard deviation of 1510). In other words, completely destroying the original hierarchy and creating a pure "Edge" organisation was optimal, but so was retaining 90% of the original hierarchical structure, as in the intermediate network of Figure 3.

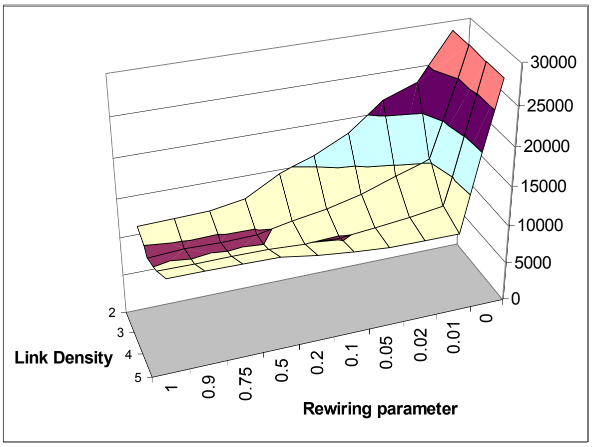

Figure 6: Average adjusted completion times plus link cost penalty for rewired networks

6. SECOND EXPERIMENT

One of the reasons that organisations are structured hierarchically is **problem decomposition**. The problem which the organisation as a whole is attempting to solve is divided into parts, which are addressed by different components of the organisation.

In order to realistically represent this phenomenon, we repeated the experiment with the agents in Figure 1 divided into three groups, corresponding to the three main branches of the structure. Instead of having all the agents solve the Lagrange problem, the three groups had three different problems:

- the Lagrange problem (for which 0, 1, 4, and 9 were potentially useful data-numbers);
- the problem of finding two data-numbers of the form $\pi - 1$ (for prime π), adding up to a positive even target-number (these exist by Goldbach's Conjecture [9], and 1, 2, 4, 6, 10, and 12 were potentially useful data-numbers);
- the problem of finding a data-number of the form $\pi + 1$ (for prime π), within the range $k + 1$ to $2k + 1$ for a target-number $k \ge 2$ (this exists by Bertrand's Postulate [10], and 4, 6, 8, 12, 14, and 18 were potentially useful data-numbers).

There is some overlap here in which data-numbers are considered potentially useful: the data-number 4 was useful for all three problems, and the data-numbers 1, 6, and 12 were each useful for two different problems. However, agents benefited from being connected to their group members, who were more likely to broadcast useful information.

For this second experiment, the adjusted completion time T was best predicted by the average distance D, with the rewiring parameter q having an independent effect. The correlation was 0.87 ($R^2 = 76\%$), and the line of best fit was:

$$T \approx 22000\,D - 2570\,D^2 + 4100\,q - 24400 \qquad (8)$$

Figure 7 shows the scatter-plot associated with this line of best fit.

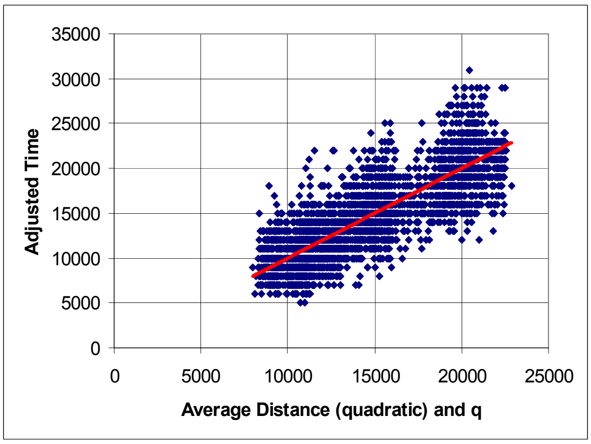

Figure 7: Scatter-plot for regression equation (8)

All the terms in this line of best fit are statistically significant (at the 0.002 level, by analysis of variance), and higher powers or other variables do not improve the prediction. Of the 76% of the variance predicted, 69% was predicted by the quadratic in D, and an additional 7% by the rewiring parameter q.

The positive coefficient for q indicates that loss of the original hierarchical structure has a negative effect. This is because of the benefit to agents of remaining connected to their fellow group members.

When we take link costs into account by adding a penalty of 60 time units per link to the adjusted completion time T, we obtain the graph in Figure 8. Here the fully rewired "Edge" structures did not perform well, and the best value for $T + 60(A + H)$ was obtained either with $4 \leq \delta \leq 5$ and $0.01 \leq q \leq 0.1$, or with $\delta = 4$ and $q = 0.2$.

There was no statistically significant difference between these cases. The average value of the cost-adjusted completion time $T + 60(A + H)$ for these cases was 19320 (with a standard deviation of 2180).

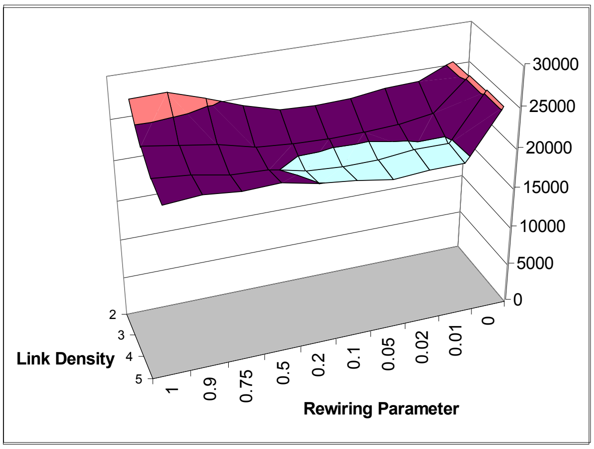

Figure 8: Average adjusted completion times plus cost for rewired networks with three different problems

The optimal combinations $4 \leq \delta \leq 5$ and $0.01 \leq q \leq 0.1$, or $\delta = 4$ and $q = 0.2$ include the network in Figure 3, but also the network with $\delta = 5$ and $q = 0.01$ in Figure 9. In that network, only one of the original hierarchical links has been lost.

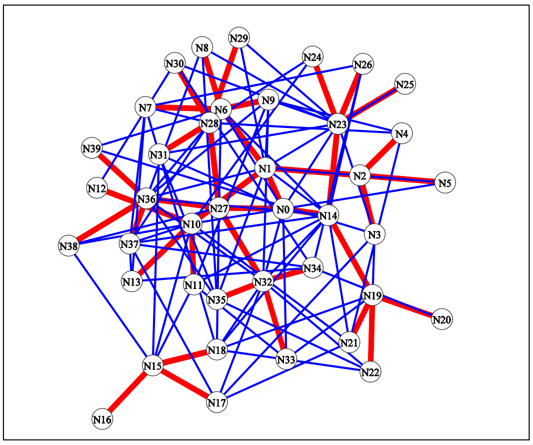

Figure 9: Cross-linked hierarchy, created with $\delta = 5$ and $q = 0.01$ (one original link has been lost)

However, although the optimal cases in this second experiment were intermediate networks, the optimum was a relatively poor one. On average, about 11 of the 40 agents failed to solve their problem, even though several other agents had the information they required to do so. The ultimate cause of this poor performance was the failure of agents to share information which they did not realise was useful to others. In the next section, we explore strategies for dealing with this problem.

7. DISCUSSION

In this paper, we have introduced a tree rewiring process (inspired by Watts [4]) which interpolates between pure hierarchical tree structures, and totally random "Edge" organisations. Our tree rewiring process is controlled by two parameters, a rewiring parameter q, and a link density δ. The rewiring parameter q controls how much of the original hierarchical structure is lost, and both parameters together control how many new communication links are added to the organisation.

We have evaluated the performance of the resulting spectrum of organisations using a simple simulation model based on mathematical problem-solving.

If we do not take into account the cost of adding new communication links, the best performance is obtained with an extreme "Edge" organisation which replaces the original hierarchy with as many new randomly-connected links as possible.

However, if we do take link costs into account, there are two optima: a pure "Edge" organisation without too many links ($\delta = 3$, $0.5 \leq q \leq 1$) and an intermediate network such as that shown in Figure 3 ($\delta = 4$, $q = 0.1$).

If we assume that the original hierarchy corresponds to a problem decomposition, so that different branches of the hierarchy are solving slightly different problems, then the optimum moves towards cross-linked hierarchies such as that in Figure 9. However, this result assumes that the different tasks carried out by different branches of the hierarchy compromise the ability or willingness of agents to share useful information.

Hierarchies are known to have problems with collaboration and information sharing [11]. As Figure 6 illustrates, "Edge" organisations have the potential to produce superior performance by addressing this problem. However, creating effective "Edge" organisations requires introducing both technological and cultural mechanisms to encourage agents to share information and collaborate. In the US model of "Edge" organisations, these are the Global Information Grid [1], and the "post before process" philosophy, which encourages agents to post information on the network before they have processed it and made a decision on its usefulness.

When technological and cultural mechanisms to encourage information-sharing and collaboration are introduced, there are two consequences:

- extreme “Edge” organisations begin to become optimal, as shown in Figure 6; and
- overall performance is improved.

This experimental investigation has considered only two of the Albert-Hayes/NATO characteristics of C2 (patterns of interaction and information dissemination), and has done so jointly. It would also be of value to experiment with varying these two characteristics independently, although they are obviously very closely linked.

The third Albert-Hayes/NATO characteristic (decision rights) has an independent effect, which is influenced by the difficulty of the problem which the organisation is attempting to solve [3],[12].

For problems of moderate difficulty, such as air strikes against stationary or slow-moving targets, highly centralised decision-making is appropriate. When the problem is easy enough, and there is sufficient time, centralised decision-making can deliver a “best possible” solution which minimises potential problems. An example of this approach is the Air Tasking Order, as used by the US Air Force, which controls individual aircraft “by tail number” from theatre level command centres, using a fixed (72-hour) planning cycle [1].

For more complex problems, particularly warfighting in the land environment, no “best possible” solution is possible within the available timeframe. Under these circumstances, self-synchronised “Edge” approaches perform best, provided the quality of networking is adequate [3],[13]. This includes both technological aspects of networking (such as bandwidth and availability), as well as human aspects (such as the time to make decisions, and the likelihood of passing on information).

The technological and human aspects of networking therefore control the choice between hierarchical and “Edge” alternatives on all three of the Albert-Hayes/ NATO C2 characteristics. These aspects of networking also control the benefits chain for Network Centric Warfare or Network Enabled Capability [14]. The experiments reported here show how successfully addressing these aspects of networking can improve organisational performance by making effective “Edge” organisations possible.

Related work by Orr and Nissen [15] supports our conclusions, finding that “Edge” organisations can outperform hierarchies, especially in challenging environments.

8. ACKNOWLEDGEMENTS

Thanks to Adam Forsyth for comments on a previous version of the tree rewiring process.

REFERENCES

1. Alberts, D.S. & Hayes, R.E. (2003) *Power to the Edge*. CCRP Press: Washington, DC. Available at www.dodccrp.org/files/Alberts_Power.pdf
2. Alberts, D.S. & Hayes, R.E. (2006) *Understanding Command and Control*. CCRP Press: Washington. Available at www.dodccrp.org/files/Alberts_UC2.pdf
3. Dekker, A.H. (2006) “Centralisation vs Self-Synchronisation: An Agent-Based Investigation,” *Proc. 11th International Command and Control Research and Technology Symposium*, CCRP Press: Washington, DC.
4. Watts, D. (2003) *Six Degrees: The Science of a Connected Age*. William Heinemann: London.
5. Dekker, A.H. (2006) “Collaborative Problem-Solving and Network Structure,” Presentation to Complex Adaptive Systems and Interacting Agents Workshop, Oxford, UK. www.acm.org/~dekker/CASIA.pdf
6. Weisstein, E.W. (2006) “Lagrange’s Four-Square Theorem,” mathworld.wolfram.com/LagrangesFour-SquareTheorem.html
7. Dekker, A.H. (2005) “Network Topology and Military Performance,” *MODSIM 2005 International Congress on Modelling and Simulation* (Zerger, A. and Argent, R.M. eds), pp 2174–2180, ISBN: 0-9758400-2-9. www.mssanz.org.au/modsim05/proceedings/papers/dekker.pdf
8. Dekker, A.H. (2004) “Simulating Network Robustness: Two Perspectives on Reality,” *SimTecT Simulation Technology and Training Conference*.
9. Weisstein, E.W. (2006) “Goldbach Conjecture,” mathworld.wolfram.com/GoldbachConjecture.html
10. Sondow, J. & Weisstein, E.W. (2006) “Bertrand's Postulate,” mathworld.wolfram.com/BertrandsPostulate.html
11. Handy, C. (1993) *Understanding Organizations*, 4th edition, Penguin: London.
12. Dekker, A.H. (2003) “Centralisation and Decentralisation in Network Centric Warfare,” *Journal of Battlefield Technology*, vol. 6, no. 2, July, pp 23–28.
13. Dekker, A.H. & de Silva, L. (2006) “Investigating Organisational Structures with Networks of Planning Agents,” in *International Conference on Intelligent Agents, Web Technologies and Internet Commerce (IAWTIC06)*, Sydney, Australia, 29 Nov – 1 Dec.
14. Court, G. (2006) “Validating the NEC Benefits Chain,” *Proc. 11th International Command and Control Research and Technology Symposium*, CCRP Press: Washington, DC.
15. Orr, R.J. & Nissen, M.E. (2006) “Hypothesis Testing of Edge Organizations: Simulating Performance under Industrial Era and 21st Century Conditions,” *Proc. 11th International Command and Control Research and Technology Symposium*, CCRP Press: Washington, DC.